

# COMPOSITE OPERATORS IN FUNCTIONAL RENORMALIZATION GROUP APPROACH

Olga V. Zyryanova <sup>a</sup>

<sup>(a)</sup> *Tomsk State Pedagogical University,  
Kievskaya St. 60, 634061 Tomsk, Russia*

## Abstract

The gauge dependence problem of the effective action for general gauge theories in the framework of a modified functional renormalization group approach proposed recently is studied. It is shown that the effective action remains gauge-dependent on-shell.

**Keywords:** Effective action, general gauge theories, gauge dependence, composite operators.

**PACS:** 11.15.-q, 11.15.Bt

*E-mail:* zyryanova@tspu.edu.ru

# 1 Introduction

The functional renormalization group approach [1, 2] is an attempt to give a method to consider some non-perturbative aspects of quantum theories. There is a rather high activity in applications of this method as to various problems in the quantum field theory (see, for example, [3, 4] and references therein). Special attention is given to the gauge theories as the ones in terms of which all fundamental interactions might be described. Recently, it has been shown that the application of the functional renormalization group approach to the Yang-Mills theories faces the gauge dependence problem as the effective action being the main object of the method and satisfying the flow equation remains gauge dependent even on-shell [4]. Moreover, the gauge dependence of the effective action has been explicitly demonstrated by explicit calculations for a simple Abelian model in linear gauges [5]. These facts tell us that there is no possibility for a reasonable physical interpretation of any results obtained within the framework of standard FRG [1, 2] being applied as to gauge systems.

Just recently, a modification of the standard FRG approach has been proposed [6]. The modification consists of a single insertion of composite operators into the generating functional of Green functions. Application of this proposal to the gauge theories requires the general study of the gauge dependence problem. In the present paper, we analyze this problem as for the general gauge theories formulated in the field-antifield quantization formalism, in arbitrary admissible gauges [7, 8]. We find that the modified FRG approach [6] suffers with the same problem as the standard one [1, 2].

We use the condensed DeWitt's notations [9]. The Grassmann parity of a quantity  $F$  is denoted as  $\varepsilon(F)$ .

## 2 Gauge dependence problem in the FRG approach

Our starting point is a gauge theory described within the framework of the field-antifield formalism [7, 8] by the effective action functional  $S_{eff}(\varphi)$  as defined by

$$S_{eff}(\varphi) = S_{ext}(\varphi, \varphi^*)|_{\varphi^*=0} = S\left(\varphi, \varphi^* + \frac{\delta\Psi}{\delta\varphi}\right)|_{\varphi^*=0}, \quad (2.1)$$

where  $\varphi = \{\varphi^A\}$  ( $\varepsilon(\varphi^A) = \varepsilon_A$ ) means the fields of the total configuration space, including the initial fields of the classical gauge theory, the ghost and the antighost fields, the Nakanishi - Lautrup fields, and so on. Here  $\varphi^* = \{\varphi_A^*\}$  ( $\varepsilon(\varphi_A^*) = \varepsilon_A + 1$ ) denotes the set of the respective antifields. Both the actions  $S_{ext} = S_{ext}(\varphi, \varphi^*)$  and  $S = S(\varphi, \varphi^*)$  satisfy the quantum master equation

$$\Delta \exp \left\{ \frac{i}{\hbar} S_{ext} \right\} = 0, \quad \Delta \exp \left\{ \frac{i}{\hbar} S \right\} = 0, \quad \Delta = (-1)^{\varepsilon_A} \frac{\overrightarrow{\delta}}{\delta\varphi^A} \frac{\overrightarrow{\delta}}{\delta\varphi_A^*}, \quad \Delta^2 = 0. \quad (2.2)$$

Fermion functional  $\Psi = \Psi(\varphi)$  describes the gauge fixing characteristic to the field-antifield formalism. In turn, the natural arbitrariness in  $S_{ext}$  is described by the formula (see, for instance, [10, 11])

$$\exp \left\{ \frac{i}{\hbar} S_{ext} \right\} = \exp \{ [\Delta, \Psi] \} \exp \left\{ \frac{i}{\hbar} S \right\}, \quad (2.3)$$

where  $[ , ]$  stands for the supercommutator,  $[F, G] = FG - GF(-1)^{\varepsilon(F)\varepsilon(G)}$ .

The generating functional of the Green functions for the general gauge theories within the FRG approach can be written down in the form of a functional integral over the fields  $\varphi$ ,

$$Z_k(J, K) = \int D\varphi \exp \left\{ \frac{i}{\hbar} [S_{eff}(\varphi) + J_A \varphi^A + S_k(\varphi) + K \mathcal{O}(\varphi)] \right\}, \quad (2.4)$$

$\mathcal{O}(\varphi)$  is an arbitrary composite operator,  $J = \{J_A\}$ ,  $(\varepsilon(J_A) = \varepsilon_A)$  are the respective sources to the fields  $\varphi = \{\varphi^A\}$ ,  $K$  is the source to the composite operator  $\mathcal{O}(\varphi)$  ( $\varepsilon(K) = \varepsilon(\mathcal{O}(\varphi))$ ). The action  $S_k(\varphi)$  named the regulator action within the FRG approach [1, 2, 3] has the quadratic form,

$$S_k(\varphi) = \frac{1}{2} R_{kAB} \varphi^B \varphi^A \equiv \int dx dy \frac{1}{2} R_{kAB}(x, y) \varphi^B(x) \varphi^A(y), \quad (2.5)$$

where  $R_{kAB}(x, y)$ ,  $R_{kAB}(x, y) = R_{kBA}(y, x)(-1)^{\varepsilon_A \varepsilon_B}$  are the so-called regulator functions. Index  $k$  denotes a momentum rescaling parameter such that

$$\lim_{k \rightarrow 0} S_k(\varphi) = 0. \quad (2.6)$$

The standard FRG approach [1, 2] corresponds to the case when  $K = 0$  while the modified one [6] works with  $K \neq 0$ . For both these approaches the gauge dependence problem is studied in a similar way.

To use the most efficient aspects of the field-antifield formalism it is convenient to introduce the extended generating functional of the Green functions,  $Z_k(J, \varphi^*, K)$ ,

$$Z_k(J, \varphi^*, K) = \int D\varphi \exp \left\{ \frac{i}{\hbar} [S_{ext}(\varphi, \varphi^*) + S_k(\varphi) + J_A \varphi^A + K \mathcal{O}(\varphi)] \right\}. \quad (2.7)$$

Obviously, one has,

$$Z_k(J, \varphi^*, K) \Big|_{\varphi^*=0} = Z_k(J, K), \quad (2.8)$$

where  $Z_k(J, K)$  is defined in (2.4).

The modified Ward identity for the generating functional  $Z_k = Z_k(J, \varphi^*, K)$  has the form

$$J_A \frac{\overrightarrow{\delta}}{\delta \varphi_A^*} Z_k + \frac{\hbar}{i} R_{kAB} \frac{\overrightarrow{\delta}^2 Z_k}{\delta J_B \delta \varphi_A^*} + K \widehat{\mathcal{O}}_A \frac{\overrightarrow{\delta}}{\delta \varphi_A^*} Z_k = 0, \quad (2.9)$$

which follows from the equality

$$\int D\varphi \exp \left\{ \frac{i}{\hbar} [S_k(\varphi) + J_A \varphi^A + K \mathcal{O}(\varphi)] \right\} \Delta \exp \left\{ \frac{i}{\hbar} S_{ext}(\varphi, \varphi^*) \right\} = 0. \quad (2.10)$$

In (2.9), the notations

$$\widehat{\mathcal{O}}_A = \mathcal{O}_A \left( \frac{\hbar}{i} \frac{\overrightarrow{\delta}}{\delta J} \right), \quad \mathcal{O}_A(\varphi) = \mathcal{O}(\varphi) \frac{\overleftarrow{\delta}}{\delta \varphi^A}, \quad (2.11)$$

are used. When  $k \rightarrow 0$  and  $K = 0$  the (2.9) reduces to the standard Ward identity playing the crucial role as to study the renormalization and the gauge dependence problem for general gauge theories within the field-antifield formalism [12].

Consider now an infinitesimal variation of the gauge fixing functional  $\Psi$  entering the action  $S_{ext}$  (2.3)

$$\Psi(\varphi) \rightarrow \Psi(\varphi) + \delta \Psi(\varphi). \quad (2.12)$$

The latter generates the following variation in  $S_{ext}$

$$\delta \exp \left\{ \frac{i}{\hbar} S_{ext} \right\} = \Delta \delta \Psi \exp \left\{ \frac{i}{\hbar} S_{ext} \right\}, \quad (2.13)$$

and, thereby, in the generating functional of the Green functions (2.7),  $Z_k = Z_k(J, \varphi^*, K)$ ,

$$\delta Z_k = \int D\varphi \exp \left\{ \frac{i}{\hbar} [S_k(\varphi) + J_A \varphi^A + K \mathcal{O}(\varphi)] \right\} \Delta \delta \Psi(\varphi) \exp \left\{ \frac{i}{\hbar} S_{ext}(\varphi, \varphi^*) \right\}. \quad (2.14)$$

By usual manipulations, the equation (2.14) rewrites in the form

$$\delta Z_k = -\frac{i}{\hbar} \left( J_A + \frac{\hbar}{i} R_{kAB} \frac{\overrightarrow{\delta}}{\delta J_B} + K \widehat{\mathcal{O}}_A \right) \delta \widehat{\Psi} (-1)^{\varepsilon_A} \frac{\overrightarrow{\delta} Z_k}{\delta \varphi_A^*}, \quad (2.15)$$

where  $\widehat{\mathcal{O}}_A$  is defined in (2.11) and

$$\delta \widehat{\Psi} = \delta \Psi \left( \frac{\hbar}{i} \frac{\overrightarrow{\delta}}{\delta J} \right). \quad (2.16)$$

By using the Ward identity (2.9), the equation defining the gauge dependence of the effective action (2.15) rewrites in a very nice form

$$\delta Z_k = \frac{i}{\hbar} [\delta \widehat{\Psi}, J_A] \frac{\overrightarrow{\delta} Z_k}{\delta \varphi_A^*} = \delta \widehat{\Psi}_A \frac{\overrightarrow{\delta} Z_k}{\delta \varphi_A^*}, \quad (2.17)$$

where

$$\delta \widehat{\Psi}_A = \delta \Psi_A \left( \frac{\hbar}{i} \frac{\overrightarrow{\delta}}{\delta J} \right), \quad \delta \Psi_A(\varphi) = \delta \Psi(\varphi) \frac{\overleftarrow{\delta}}{\delta \varphi^A}. \quad (2.18)$$

It follows from (2.15) that the relation holds

$$\delta\widehat{\Psi}_A \frac{\overrightarrow{\delta} Z_k}{\delta\varphi_A^*} \Big|_{J=0, K=0} \neq 0. \quad (2.19)$$

The relation (2.19) allows one to reveal the gauge dependence of the average effective action in the both FRG approaches, even on-shell. This means that there is no consistent physical description of the results obtained within the framework of the standard FRG approach [1, 2], or of the modified one [6], in the case of gauge theories.

### 3 Discussion

Notice that there exists a way to solve the gauge dependence problem appearing in the functional renormalization group approach with use the concept of quantum field theory with composite operators [13, 14, 15]. Following the paper [4] one can extend the action in the functional integral (2.4) with the new term,  $K_1 L_k(\phi)$  instead of  $S_k(\varphi)$ , introducing the new external source  $K_1$  as for the respective quantity  $L_k(\phi)$ . Then one can state that the effective action with these composite operators does not depend on gauges on-shell which is defined with respect to the equations of motion as for the effective action.

Quite recently the problem of modification of the standard quantization rules not destroying the gauge status of the scheme was discussed in the paper [16]. It was proven that defining the physical observable quantities,  $\mathcal{O}_{phys}$ , in the usual way, one extends the action in the functional integral with the new term  $\frac{\hbar}{i} \ln \mathcal{O}_{phys}(\phi)$ . There is no new external sources introduced in the latter case. It follows that the standard quantum master equation absorbs consistently the new term, provided that the  $\mathcal{O}_{phys}$  is annihilated by the BRST operator  $\sigma =: \frac{\hbar}{i} \Delta + \text{ad}(S_{eff})$ , where  $\text{ad}(S_{eff})$  means the adjoint antibracket operator. Then it follows that the total effective action is gauge independent on-shell. From this point of view the gauge dependence problem in the standard FRG [1, 2] is related with the fact that  $\sigma \exp\{\frac{i}{\hbar} S_k\} \neq 0$ .

### Acknowledgments

The author thanks P.M. Lavrov for useful discussions. The work is supported by the Ministry of Education and Science of Russian Federation, grant 3.1386.2017 and by the RFBR grant 15-02-03594.

## References

- [1] C. Wetterich, *Average Action And The Renormalization Group Equations*. Nucl. Phys. B352 (1991) 529.
- [2] C. Wetterich, *Exact evolution equation for the effective potential*, Phys. Lett. B301 (1993) 90.
- [3] H. Gies, *Introduction to the functional RG and applications to gauge theories*, Lect. Notes Phys. 852 (2012) 287.
- [4] P. M. Lavrov, I. L. Shapiro, *On the functional renormalization group approach for Yang-Mills fields*, JHEP **06** (2013) 086.
- [5] P. M. Lavrov, B. S. Merzlikin, *Loop expansion of average effective action in functional renormalization group approach*, Phys. Rev. D **92** (2015) 085038.
- [6] C. Pagani, M. Reuter, *Composite Operators in Asymptotic Safety*, arXiv:1611.06522 [gr-qc].
- [7] I. A. Batalin, G. A. Vilkovisky, *Gauge algebra and quantization*, Phys. Lett. B **102** (1981) 27.
- [8] I. A. Batalin, G. A. Vilkovisky, *Quantization of gauge theories with linearly dependent generators*, Phys. Rev. D **28** (1983) 2567.
- [9] B. S. DeWitt, *Dynamical Theory of Groups and Fields*, Gordon and Breach, New York, 1965.
- [10] I. A. Batalin, K. Bering, *Gauge Independence in a Higher-Order Lagrangian Formalism via Change of Variables in the Path Integral*, Phys. Lett. B **742** (2015) 23.
- [11] I. A. Batalin, P. M. Lavrov, I. V. Tyutin, *Finite anticanonical transformations in fieldantifield formalism*, Eur. Phys. J. C **75** (2015) 270.
- [12] B. L. Voronov, P. M. Lavrov, I. V. Tyutin, *Canonical transformations and the gauge dependence in general gauge theories*, Sov. J. Nucl. Phys. **36** (1982) 292.
- [13] C. De Dominicis, P. C. Martin, *Stationary entropy principle and renormalization in normal and superfield systems. I. Algebraic formulation*, J. Math. Phys. **5** (1964) 14.
- [14] J. M. Cornwall, R. Jackiw, E. Tomboulis, *Effective action for composite operators*, Phys. Rev. D **10** (1974) 2428.
- [15] P. M. Lavrov, *Effective action for composite fields in gauge theories*, Theor. Math. Phys. **82** (1990) 282.
- [16] I. A. Batalin, P. M. Lavrov, *Physical quantities and arbitrariness in resolving quantum master equation*, arXiv:1702.02663 [hep-th].